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terms in the lunar theory, especially the parallactic inequality, are affected by instrumental errors.

(*c*) The Moon's longitude determined by the old meridian instruments differs from that by the altazimuth, *after allowing for different apparent diameter*, by a *variable* quantity which rises to $-0''\cdot7$ at about the quarters compared with full moon, and is represented in the fourth column of Table V.

(*d*) The new transit circle, on the other hand, shows an increasing positive difference, slowly increasing even past the quarters (see the fifth column of Table V.)

(*e*) The evidence is against this change being due to any change in the altazimuth.

(*f*) It therefore seems probable that meridian instruments may give very different values for the parallactic inequality; and the results derived by Mr. Cowell on pp. 95–98 of *Monthly Notices*, vol. lxiv., especially that for the Sun's parallax, must, until these instrumental errors have been more fully discussed, be received with caution.

University Observatory, Oxford:
1904 *February* 20.

Methods of Analysis of Moon's Errors and some Results.

By P. H. Cowell, M.A.

I have had a very short glance at Mr. Nevill's paper * in MS. before it was sent to the printers, and in consequence I am now giving a full account of my methods, as my recollection is that Mr. Nevill misunderstands me in a few points. I gladly take the opportunity of acknowledging the assistance that Mr. Nevill's paper is sure to be to me, and I must also admit two or three errors in my paper that Mr. Nevill points out. Nothing in my investigations at present has turned upon these errors, but it is naturally of service to me to have them pointed out. I may add that as regards the term in cos ☊ I followed Airy too rashly. He expressly states that this term is not required by theory.

My method briefly is to analyse the observed errors. I naturally use the theory to suggest periods, and as far as possible to interpret the results, but my method is essentially empirical. I hope to obtain from the observations all inequalities in the errors of Hansen's tables exceeding $0''\cdot5$, except, perhaps, in a few instances of terms of very long period, where such accuracy is probably unattainable. A term of less than $0''\cdot5$ will probably not be detected, but coefficients of terms examined should in

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* It has been found necessary to send the proofs of Mr. Nevill's paper to the author for correction. The paper cannot therefore appear in the present Number. Professor Brown's paper is also deferred for a similar reason.

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general be obtained from fifty-four years' observations correctly to $0''\cdot1$ or less. There are two essential points in such an analysis, both of them well known. The first admits of no variation in its method of treatment. The second I treat in a somewhat novel fashion, which I shall endeavour to explain.

The first of these points is the near equality of many of the periods. Let us, for example, consider the inequalites with arguments nearly equal to the mean anomaly g. Let them be denoted by $\Sigma_s a_s \sin(g+\theta_s)$, where the quantities θ denote angles of very long period (differing from suffix to suffix). The observations are analysed into the form for each period of analysis—

$$\text{Error} = (\text{coefficient of } \sin g) \times \sin g + (\text{coefficient of } \cos g) \times \cos g$$

It is then attempted to equate

$$(\text{coefficient of } \sin g) \text{ to } \Sigma a_s \cos \theta_s$$

and

$$(\text{coefficient of } \cos g) \text{ to } \Sigma a_s \sin \theta_s$$

The quantity θ_s is obtained approximately from the observations, and then when possible identified with some argument suggested by theory. The coefficient a_s is obtained by solving the equations.

Investigations usually proceed on these lines. Certainly Professor Newcomb's (1876), Mr. Nevill's (1884), and Mr. Bakhuyzen's (1903) do. I have departed in one small point only from the practice established by these writers. My period of analysis is 400 lunar days instead of one year. My reasons for this are: (i.) 400 lunar days is fifteen periods of the mean anomaly, fourteen of the parallactic inequality, and thirteen of the evection; it is also a round number; (ii.) my plan is to remove all new inequalities from the tabular places, and to effect this speedily I have arranged the observations in columns of forty lines each. Each line of every column corresponds to a transit of the Moon, whether observed or not. This arrangement greatly facilitates the analysis as well as the application of corrections. It obviously requires the period of analysis to contain an exact number of columns.

The second point depends upon the want of symmetry in the distribution of the observations with reference to the age of the Moon. It is evident that, if the Moon was only observed when it was full, a real inequality $a \sin \phi$ could not be distinguished from apparent inequalities $-a \sin(\phi+\mathrm{D})$ and $-a \sin(\phi-\mathrm{D})$. In my paper in the December number of the *Monthly Notices* I showed that in 5647 observations the mean value of $\cos \mathrm{D}$ is $-2721 \div 5647 = -0\cdot48$, and of $\sin \mathrm{D} + 416 \div 5647 = +0\cdot07$. The latter quantity may be neglected, but from the former I infer the rough rule that a real inequality $a \sin \phi$ will give rise to two apparent inequalities $-\frac{1}{2}a \sin(\phi+\mathrm{D})$ and $-\frac{1}{2}a \sin(\phi-\mathrm{D})$. Conversely, given an apparent inequality $\beta \sin \phi$, it is not certain

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that such a term is required by the tables. The term actually required may be $-2\beta \sin(\phi+D)$ or $-2\beta \sin(\phi-D)$. The terms $\phi \pm D$ must be examined before the nature of the apparent inequality can be determined. Below, when I come to the results, a few instances will be given where a second analysis (after removing some terms) has amply justified the method of reasoning.

n observations are considered to indicate an apparent inequality $\beta \sin \phi$, where

$$\beta = (\Sigma \epsilon \sin \phi)/\tfrac{1}{2}n$$

ϕ being any argument, of period short compared with the period of analysis, and not either equal, or nearly equal, to D, the age of the Moon, and ϵ being the observed error.

It will be seen below that ϕ has been put equal to g, the mean anomaly, $2D-g$, the evection, and subsequently g a second time after the removal of some terms, the period of analysis being 400 lunar days, and 48 such periods being used. Also ϕ has been put equal to g', the Sun's mean anomaly, and $g-D$, an argument whose period is 400 lunar days, the whole 48×400 lunar days being in these cases taken as a single period of analysis.

In the December *Monthly Notices*, at the beginning of my paper on the semi-diameter &c., I mentioned that the tabular places from 1847 to 1901 had been corrected, when necessary, for Newcomb's corrections and for Hansen's term with wrong sign. Previous to the investigation of that paper they were also corrected for five other terms, making seven corrections in all. They have subsequently been corrected for

(8) semi-diameter (correction $0''.4$ in accordance with the results of my paper, this being a reasonable approximation to $\pm 0''.346 + 0''.120 \sin D$).

(9) The long-period terms were taken out approximately by subtracting suitable constants from all the errors throughout a whole period of analysis.

This may be looked upon as equivalent to equating each error to $c+\beta \sin \phi$, instead of $\beta \sin \phi$ only. The normal equation for β will then contain c with a small coefficient; as an approximate value for c, the mean error of the period may clearly be taken. Since in some periods of analysis c exceeds $6''$, it must be more accurate to take account of it. The error of omitting it is $\frac{2c}{n} \Sigma \sin \phi$, and $\frac{1}{n} \Sigma \sin \phi$ is liable to be about 0·1. By this method the error is reduced to $\frac{2\Delta c}{n} \Sigma \sin \phi$, Δc being the quantity by which the estimate of c differs from the truth. When the observations happen to be well distributed as regards the argument ϕ, $\Sigma \sin \phi$ will be nearly zero.

(10) $-0''.246 \sin (2D-g)$

a correction to the evection.

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(11) $+0''{\cdot}316 \sin (g+2\varpi-3J+7^\circ)$

a term due to the action of *Jupiter.*

(12) $+0''{\cdot}55 \sin g'$

a correction to the annual equation.

(13) $-0''{\cdot}55 \sin (g-D)$,

a correction to Hansen's tables that I have discovered by analysis, but which is confirmed by the revised results of the *Darlegung.*

The following table gives the results of the different analyses.

Coefficients for 48 *periods of analysis of* 400 *lunar days each of terms in the tabular minus observed errors of longitude in tenths of a second of arc.*

	cos g.	sin g.	cos $(2D-g)$.	sin $(2D-g)$.	cos g.	sin g.
86	− 4	− 1	+ 4	+ 6	− 3	− 1
87	− 3	− 10	+ 2	+ 10	− 1	− 7
88	− 18	− 13	0	+ 8	− 15	− 4
89	− 24	− 6	− 5	+ 9	− 19	0
90	− 14	+ 9	+ 2	+ 2	− 12	+ 6
91	− 18	+ 9	+ 1	+ 2	− 13	+ 6
92	− 2	+ 7	− 1	+ 6	− 1	+ 12
93	+ 5	− 5	+ 1	+ 8	+ 5	− 1
94	+ 9	− 9	0	+ 11	+ 3	+ 2
95	− 12	− 11	− 1	+ 10	+ 1	− 6
96	− 22	− 20	− 6	+ 4	− 13	− 9
97	− 21	+ 2	− 12	+ 6	− 9	− 2
98	− 18	+ 6	− 6	− 1	− 13	− 6
99	+ 4	+ 4	− 7	− 6	− 9	+ 7
100	+ 7	− 1	− 1	− 4	+ 6	+ 5
101	+ 8	0	− 3	0	+ 1	+ 2
102	− 3	− 7	+ 1	0	+ 1	− 2
103	0	− 6	+ 2	+ 1	− 1	− 6
104	− 5	− 11	+ 4	+ 4	− 6	− 3
105	− 8	0	− 7	+ 6	− 10	0
106	− 9	0	− 6	+ 6	− 8	+ 4
107	0	+ 10	− 1	+ 8	0	+ 7
108	+ 6	+ 13	− 2	+ 4	+ 4	+ 11
109	+ 5	+ 5	− 1	+ 6	+ 1	+ 5
110	+ 7	− 1	+ 2	+ 5	+ 4	− 1
111	+ 13	+ 1	− 7	+ 13	+ 9	+ 4
112	− 2	0	− 4	+ 12	0	0
113	− 1	+ 4	− 3	+ 10	− 1	− 1
114	− 3	+ 15	− 1	+ 2	+ 3	+ 9
115	+ 5	+ 17	− 4	+ 3	+ 7	+ 14

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	cos *g*.	sin *g*.	cos (2D−*g*).	sin (2D−*g*).	cos *g*.	sin *g*.
116	+10	+ 4	− 8	+ 3	+10	+ 3
117	+ 6	+ 3	+ 3	0	+ 7	+ 1
118	+ 9	− 3	+ 1	+ 7	+11	0
119	− 9	−15	0	+10	− 7	− 9
120	−19	− 5	0	+	−13	− 2
121	−14	+ 3	− 2	+10	− 7	+ 5
122	−11	+ 9	− 2	+10	− 6	+11
123	− 7	+ 6	− 1	+ 9	− 3	+ 7
124	+ 3	+ 8	− 7	+ 4	+ 4	+11
125	+11	− 4	− 4	− 2	+14	+ 1
126	+10	−10	− 5	+ 9	+ 9	− 5
127	+ 3	−15	− 7	+14	+ 6	− 6
128	− 6	−13	− 6	+ 5	− 3	− 5
129	− 9	+ 1	− 3	+ 8	− 6	+ 6
130	− 2	− 4	− 6	+ 3	0	+ 3
131	+ 3	+ 4	− 4	− 3	+ 1	+ 7
132	+ 9	− 3	− 6	+ 2	+ 7	+ 6
133	+ 7	−14	− 3	+ 5	+ 1	− 3
Means	−0″·26	−0″·10	−0″·25	+0″·52	−0″·13	+0″·16

The first two columns (cos *g* and sin *g*) in the table were derived from the errors after correction (8) and before correction (9). They are liable therefore to moderately large accidental errors in certain periods proportional to $\Sigma \cos g$ and $\Sigma \sin g$ respectively.

The third and fourth columns (cos (2D—*g*) and sin (2D—*g*)) were derived after correction (9) and before correction (10).

The fifth and sixth columns (cos *g* and sin *g*) were derived after correction (13).

In addition to the above, by including all the observations into one single period of analysis, after correction (9), I have obtained the following *apparent* corrections. The significance of the word *apparent* will be evident from the introductory remarks to this paper :

$$\begin{aligned} &+0''\cdot59 \sin(g-D)+0''\cdot34 \cos(g-D) \\ &-0''\cdot83 \sin g'-0''\cdot33 \cos g' \\ &+0''\cdot40 \sin(\omega-\omega')-0''\cdot35 \cos(\omega-\omega') \end{aligned}$$

(The notation is Hansen's, with the additional symbol $D=g-g'+\omega-\omega'$.)

The coefficients of cos *g* and sin *g* (first set) have been analysed for planetary terms.

Since in the case of planetary terms there is no question of arguments differing by D, the distinction between real and apparent corrections does not arise.

$$\text{Coefficient of } \cos g = -0''\!\cdot\!21 \sin(2\varpi - 3J + 7^\circ) + 0''\!\cdot\!37 \sin(2\varpi - 2J) - 0''\!\cdot\!17 \cos(2\varpi - 2J)$$

$$\text{Coefficient of } \sin g = -0''\!\cdot\!52 \cos(2\varpi - 3J + 7^\circ) + 0''\!\cdot\!02 \sin(2\varpi - 2J) + 0''\!\cdot\!09 \cos(2\varpi - 2J)$$

Combining the two terms and omitting small terms we obtain

$$\text{(from periods 86–133)} \quad -0''\!\cdot\!36 \sin(g + 2\varpi - 3J + 7^\circ) + 0''\!\cdot\!23 \sin(g + 2\varpi - 2J)$$

In a similar manner, using the values of a and b published in the November *Monthly Notices*, we obtain

$$\text{(from periods 1–48)} \qquad +0''\!\cdot\!20 \sin(g + 2\varpi - 2J)$$

$$\text{(from periods 42–89)} \qquad +0''\!\cdot\!28 \sin(g + 2\varpi - 2J)$$

It appears therefore that the tabular places require the correction

$$+0''\!\cdot\!36 \sin(g + 2\varpi - 3J + 7^\circ) - 0''\!\cdot\!23 \sin(g + 2\varpi - 2J)$$

The first term agrees closely with theory, for Radau gives $+0''\!\cdot\!316$ as the coefficient.

The second term indicates a correction $-0''\!\cdot\!23$ to the $-0''\!\cdot\!893$ already applied. This term is of especial interest, for it will be remembered that it was discovered empirically by Professor Newcomb in 1876, who assigned it a coefficient $1''\!\cdot\!5$. Mr. Neison first pointed out that the term is due to the action of *Jupiter*. It appears from this paper that the coefficient given by the observations is $1''\!\cdot\!1$. Hill and Radau both obtain $0''\!\cdot\!9$ from theory.

Coming now to the solar terms, I obtain the following *apparent* terms in the tabular *minus* observed errors :

$$-0''\!\cdot\!52 \sin(g - 2D) - 0''\!\cdot\!25 \cos(g - 2D)$$
$$+0''\!\cdot\!59 \sin(g - D) + 0''\!\cdot\!34 \cos(g - D)$$
$$-0''\!\cdot\!10 \sin g - 0''\!\cdot\!26 \cos g$$

Some more terms are given below after the discussion of the above. The coefficients of $\sin g$ and $\cos g$ are from the first analysis.

The cosine terms, if real, denote an error of epoch. Clearly no change of the epoch of g will explain all three terms. The terms are, however, small. I hold over the discussion until the tabular places from 1750 to 1851 have been revised and the observations discussed.

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The three sine terms suggest the real corrections

$$+0''{\cdot}25 \sin(g-2D) - 0''{\cdot}55 \sin(g-D) - 0''{\cdot}17 \sin g$$

are required by the tabular places, for a real correction of this form will produce an apparent correction

$$\begin{gathered}(+0''{\cdot}25+0''{\cdot}27)\sin(g-2D)\\(-0''{\cdot}12-0''{\cdot}55+0''{\cdot}08)\sin(g-D)\\(+0''{\cdot}27-0''{\cdot}17)\sin g\end{gathered}$$

on the principles explained above, thus annihilating the terms indicated by the errors with an accuracy that is almost suspicious.

It will be noticed that the terms $+0''{\cdot}25 \sin(g-2D) - 0''{\cdot}55 \sin(g-D)$ have been applied, and the second analysis gives for the tabular minus observed errors $+0''{\cdot}17 \sin g$. This second analysis bears out in a remarkable degree the substantial accuracy of the assumption that a real correction $\beta \sin \phi$ produces apparent corrections

$$\beta \sin \phi - \tfrac{1}{2}\beta \sin(\phi+D) - \tfrac{1}{2}\beta \sin(\phi-D)$$

The explanation of these corrections is interesting. The coefficient of the evection in Hansen's tables is 4586″·690 (see January *Monthly Notices*) ; the true coefficient is therefore 4586″·44. The theoretical coefficient for an elliptic coefficient 22639″·5 is about 4586″·34. The difference 0″·10 is either accidental or points to an ellipticity in the figure of the Moon only one-sixth of what Hansen assumed. It may be recalled that my paper in the December *Monthly Notices* on the evidence of the variation tended to support Hansen's hypothesis ; but there can now be little doubt that the ellipticity of the Moon is insensible or nearly so, and that the apparent increase of the variation is due to the conditions of observation, as Professor Newcomb has pointed out.

I did not expect to find so large a correction as $-0''{\cdot}55 \sin(g-D)$ in a term where Hansen's *Darlegung* and Delaunay are in such close accordance. The coefficients given by these two authorities are respectively 18″·54 and 18″60 when reduced to a parallax 8″·76, and when Delaunay's series is completed by induction. The difficulty is explained, however, by the fact that Hansen's tables differ considerably from the *Darlegung* in this term. The coefficient in the tables is 19″·15, and the correction −0″·55 might therefore have been anticipated.

The value of the principal elliptic coefficient now appears to be

$$22640''{\cdot}17 - 0''{\cdot}55 - 0''{\cdot}17 = 22639''{\cdot}45$$

on the assumption that the terms in $g \pm D$ are

$$-8''{\cdot}54 \sin(g+D) + 18''{\cdot}60 \sin(g-D).$$

As, however, the former term apparently should be

$$-8''{\cdot}46 \sin (g+\mathrm{D})$$

the principal elliptic coefficient should be

$$22639''{\cdot}49$$

My preliminary discussion of Airy's period 1750–1851 if reduced to the same terms

$$-8''{\cdot}46 \sin (g+\mathrm{D})+18''{\cdot}60 \sin (g-\mathrm{D})$$

gives $22639''{\cdot}53$

a substantial agreement. The result, however, cannot be considered definite until Professor Brown has determined the two "allied" coefficients.

It will be noticed that all the coefficients given in the principal table of this paper show a periodicity in eight periods of analysis or nine years. This period is to be identified with $\varpi-\varpi'$ or $\omega-\omega'$. Solving on this assumption, I obtain for the tabular *minus* observed errors

$$
\begin{aligned}
&+0''{\cdot}24 \sin (g-2\mathrm{D}+\omega-\omega')\\
&-0''{\cdot}83 \sin (g-\mathrm{D}+\omega-\omega')-0''{\cdot}33 \cos (g-\mathrm{D}+\omega-\omega')\\
&+0''{\cdot}81 \sin (g+\omega-\omega')+0''{\cdot}46 \cos (g+\omega-\omega')
\end{aligned}
$$

the last line referring to the first analysis for g after correction (8). The argument of the second line is g', the Sun's mean anomaly.

The above apparent corrections seem to point to true corrections

$$
\begin{aligned}
&+0''{\cdot}55 \sin g'\\
&-0''{\cdot}57 \sin (g+\omega-\omega')-0''{\cdot}46 \cos (g+\omega-\omega')
\end{aligned}
$$

being required.

It will be seen that I have applied $+0''{\cdot}55 \sin g'$. As far as I can make out only half this term is not required by theory, but perhaps Professor Brown's theory when completed will verify the whole. This at present seems unlikely; but as to the reality of the term, I consider the evidence immensely strengthened by the corroboration derived from the "allied" term $g'-\mathrm{D}$.

I have not applied corrections

$$-0''{\cdot}57 \sin (g+\omega-\omega')-0''{\cdot}46 \cos (g+\omega-\omega')$$

because of the extreme uncertainty of their real nature. No cosine term is required by theory, and the coefficient of the sine

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term appears to require theoretically an alteration of only $0''.27$. This suggests that for the outstanding part the period may not be $g+\omega-\omega'$, but some slightly different period. Two hypotheses have occurred to me: one is that the argument should be $g+\varpi$ the Moon's longitude, or that the term is due to systematic errors of observation (since ϖ' is practically a constant, analysis of the errors cannot possibly distinguish between $g+\varpi$ and $g+\varpi-\varpi'$); the other hypothesis is that the argument is $g+2\varpi+3V-5E$, and that there is something wrong with the theoretical coefficient of the *Venus* term. The argument $g+2\varpi+3V-5E$ moves more slowly than the argument $g+\omega-\omega'$ by $4^\circ.22$ in 400 lunar days, or by one revolution in a century. Hence, fifty years' observations cannot distinguish between the two arguments, and the point must be held over until the Airy period 1750–1851 is re-discussed.

A comparison of my January paper with Professor Newcomb's transformation of Hansen's theory and comparison with Delaunay shows that the following corrections are still required in the tabular places:

$$\begin{aligned}
&-0''.27 \sin(g+\omega-\omega')\\
&+0.40 \sin(g-g')\\
&-0.45 \sin(g+2g')\\
&-0.29 \sin(g'-2\omega+2\omega')\\
&+0.27 \sin(g-g'+2\omega-2\omega')\\
&-0.26 \sin(g+2\omega)\\
&-0.72 \sin(\omega-\omega')\\
&-0.22 \sin(g+g'-\omega+\omega')\\
&-0.32 \sin(2g-3g'+4\omega-4\omega')
\end{aligned}$$

The first term has just been alluded to; the coefficient of the seventh term seems too large, judging from the observations; as to the sixth term I have estimated that corrections

$$-0''.14 \sin(g+2\omega)+0''.12 \cos(g+2\omega)$$

are required by the observations. The agreement is not very close, but the analysis can only be approximate in view of the large movement of 2ω in one period of analysis. The fourth term is "allied" to the argument $g-\omega+\omega'$, and at present my analysis throws no light upon any of the other terms.

I conclude this paper with giving the necessary data for identifying my periods of analysis with the calendar, and for obtaining the values of the arguments at various epochs. The last two lines refer to Hansen's empirical *Venus* terms.

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Values of various Arguments in the middle of the 44*th period of analysis and their movements in* 400 *lunar days or one period of analysis.*

Argument. Calendar	Value. 1800 Jan. $3^{\mathrm{d}}\cdot 6$ G.M.T.	Movement. 414·02 Solar Days.
g	159°·7658	5409°·1681
g'	4·1375	408·0583
ω	192·7447	68·0478
ω'	246·4312	21·9435
☊	33·0749	−21·9241
D	101·9418	5047·2141
$g-$D	57·8240	361·9540
$\varpi=\omega+$☊	225·8196	46·1237
$\varpi'=\omega'+$☊	279·5061	0·0194
$\varpi-\varpi'=\omega-\omega'$	306·3135	46·1043
Venus V	151·2198	663·3141
Earth E	103·6435	408·0619
Mars M	234·5551	216·9601
Jupiter J	82·1455	34·4014
$2\varpi-2$J	287·3482	23·4445
$2\varpi-3$J$+7°$	212·2027	−10·9570
$2\varpi+3$V-5E	27·0811	41·8802
A$+30°$	213·8946	1·4953
8V$-$13E$+274°\ 14'$	136·6262	1·7081

Comparisons of the Geocentric Places of the Sun and Major Planets calculated from the Tables of the American Ephemeris Office, with their Places calculated from Le Verrier's Tables, for the year 1906. By A. M. W. Downing, D.Sc., F.R.S.

This paper is supplementary to those published in *Monthly Notices*, vol. lix. No. 10, and vol. lxii. No. 1.

The American Tables of the Sun, *Mercury*, *Venus*, *Mars*, *Uranus*, and *Neptune* are due to Professor Newcomb; those of *Jupiter* and *Saturn* to Mr. G. W. Hill. These Tables are adopted in calculating the ephemerides given in the *Nautical Almanac* for 1906. The corresponding places in the *Connaissance des Temps* for the same year are from Le Verrier's Tables.

The comparisons have been made by reducing the quantities given in the *Nautical Almanac* from Greenwich noon to Paris noon at convenient intervals throughout the year, and then finding the differences between the reduced quantities and the

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